

Problem 2.52

[Difficulty: 3]

2.52 A block that is a mm square slides across a flat plate on a thin film of oil. The oil has viscosity μ and the film is h mm thick. The block of mass M moves at steady speed U under the influence of constant force F . Indicate the magnitude and direction of the shear stresses on the bottom of the block and the plate. If the force is removed suddenly and the block begins to slow, sketch the resulting speed versus time curve for the block. Obtain an expression for the time required for the block to lose 95 percent of its initial speed.

Given: Block sliding on oil layer

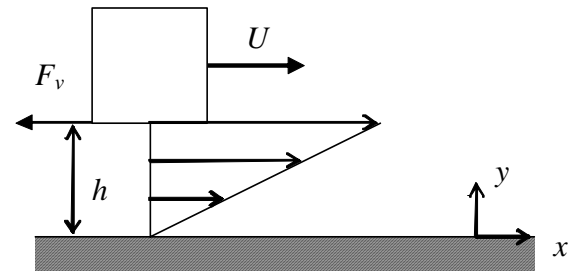
Find: Direction of friction on bottom of block and on plate; expression for speed U versus time; time required to lose 95% of initial speed

Solution:

Governing equations: $\tau_{yx} = \mu \cdot \frac{du}{dy}$ $\Sigma F_x = M \cdot a_x$

Assumptions: Laminar flow; linear velocity profile in oil layer

The bottom of the block is a $-y$ surface, so τ_{yx} acts to the left; The plate is a $+y$ surface, so τ_{yx} acts to the right



Equation of motion $\Sigma F_x = M \cdot a_x$ so $F_v = M \cdot \frac{dU}{dt}$

The friction force is $F_v = \tau_{yx} \cdot A = \mu \cdot \frac{du}{dy} \cdot A = \mu \cdot \frac{U}{h} \cdot a^2$

Hence $-\frac{\mu \cdot a^2}{h} \cdot U = M \cdot \frac{dU}{dt}$

To solve separate variables $\frac{1}{U} \cdot dU = -\frac{\mu \cdot a^2}{M \cdot h} \cdot dt$

$$\ln\left(\frac{U}{U_0}\right) = -\frac{\mu \cdot a^2}{M \cdot h} \cdot t$$

Hence taking antilogarithms $U = U_0 \cdot e^{-\frac{\mu \cdot a^2}{M \cdot h} \cdot t}$

Solving for t $t = -\frac{M \cdot h}{\mu \cdot a^2} \cdot \ln\left(\frac{U}{U_0}\right)$

Hence for $\frac{U}{U_0} = 0.05$ $t = 3.0 \cdot \frac{M \cdot h}{\mu \cdot a^2}$

